Predicate logic as a formal language

The first thing to note is that there are two sorts of things involved in a predicate logic formula. The first sort denotes the objects that we are talking about: individuals such as a and p (referring to Andy and Paul) are examples, as are variables such as x and v. Function symbols also allow us to refer to objects: thus, m(a) and g(x, y) are also objects. Expressions in predicate logic which denote objects are called terms.

The other sort of things in predicate logic denotes truth values; expressions of this kind are formulas: Y (x, m(x)) is a formula, though x and m(x) are terms.

A predicate vocabulary consists of three sets: a set of predicate symbols P, a set of function symbols F and a set of constant symbols C. Each predicate symbol and each function symbol comes with an arity, the number of arguments it expects. In fact, constants can be thought of as functions which don't take any arguments (and we even drop the argument brackets) – there- fore, constants live in the set F together with the 'true' functions which do take arguments. From now on, we will drop the set C, since it is convenient to do so, and stipulate that constants are 0-arity, so-called nullary, functions.

Terms

The terms of our language are made up of variables, constant symbols and functions applied to those. Functions may be nested, as in m(m(x)) or g(m(a), c): the grade obtained by Andy's mother in the course c.

Definition

- Any variable is a term.
- If c ∈ F is a nullary function, then c is a term.
- If t1, t2,...,tn are terms and f ∈ F has arity n > 0, then f(t1, t2,...,tn) is a term.
- Nothing else is a term.

In Backus Naur form we may write

t ::= x | c | f(t,...,t)

where x ranges over a set of variables var, c over nullary function symbols in F, and f over those elements of F with arity n > 0. It is important to note that

- the first building blocks of terms are constants (nullary functions) and variables;
- more complex terms are built from function symbols using as many previously
- built terms as required by such function symbols; and the notion of terms is dependent on the set F. If you change it, you change the set of terms.